WHAT'S THE DEAL WITH FRACTIONS

Tim Sibbald
Schulich School of Education, Nipissing University

Session Description

◦ During my session at OCMA2020 the issue was raised that students routinely struggle with fractions. Why is that? There has been considerable research regarding fractions that clarifies a variety of issues. This presentation will address recent curriculum changes with respect to fractions and the larger K–12 place of fractions. The implications about fractions for college instruction and students heading into careers are not conclusive, but there are a number of strategy implications.

Organization

◦ Identify some issues

◦ Establish the context for learners

◦ What is changing with teachers?

◦ Research

◦ What options colleges have

Zoom Details:

• Periodically, I will ask for questions
• You may also place question in the chat
• There will also be time for discussion at the end

Why people don't talk about fractions

◦ In my experience, raising the topic has:

  ◦ Wide recognition of issues

  ◦ Blamed the school system

  ◦ Claimed it was better twenty years ago in the school system

  ◦ Been highly opinionated (and somewhat polarized)

    ◦ Opinions defy daily usage or needs

  ◦ Decried the depth of consequences

  ◦ Often have added requirements ("without a calculator")

◦ Sometimes

  ◦ "Silver bullet" solutions – usually disproportionate

    ◦ "Get rid of phys ed and teach more math" (??)

    ◦ "Fire all elementary teachers and replace with strong math teachers" (??)

www.Alibaba.com

What are the Issues?

- Students routinely struggling with use of fractions
  - Unsure of algorithms, not sure where to start
  - Unclear of accuracy or interpretation
- Reluctance to use fractions
  - Recognition of the issue
  - They know what they don't know, and are responding to it.

- Reliance on calculators
  - Fractions in, decimals out
  - Not using "fraction mode"
- Usage is important in some careers
  - Dosing - nursing, particularly pediatrics
  - Mixing – art to construction

K – 8 (Using 2020 curriculum)

- Relatively stable over the years, but some alterations
- G1-3 Emphasis on fractions with small denominators, meaning of fractions in terms of fairness, equivalent fractions (1/3=2/6 Grade 2)
- G4-6 Denominators to 12, proper and improper, various contexts, equivalence of fractions and decimals, add/subtract like (G5) then unlike (G6) denominators, mult/div fraction by whole
- G7-8 Interchangeability of fractions, decimals, and percents, ratios and rates, add/subtract using equivalent fractions, mult/div fractions by fractions, fractions as proportionality

Strand B. Number	Strand C. Algebra	Strand D. Data	Strand E. Spatial Sense	Strand F. Financial Literacy
B1. Number Sense • whole numbers • rational and irrational numbers • fractions, decimals, and percents **B2. Operations** • properties and relationships • math facts • mental math • addition and subtraction • multiplication and division	**C1. Patterns and Relations** • patterns **C2. Equations and Inequalities** • variables and expressions • equalities and inequalities **C3. Coding** • coding skills **C4. Mathematical Modelling**	**D1. Data Literacy** • data collection and organization • data visualization • data analysis **D2. Probability**	**E1. Geometric and Spatial Reasoning** • geometric reasoning • location and movement **E2. Measurement** • attributes • length • mass, capacity and volume • area and surface area • angles • time • the metric system	**Grades 1 to 8: F1. Money** • money concepts **Grades 4 to 8: F1. Finances** • financial management • consumer and civic awareness

High School

◦ New Grade 9 – Fraction skills

◦ Otherwise (includes old Grade 9)

  ◦ No explicit fraction specific skills

  ◦ Implicit implication that improper can be used exclusively ("fractional coefficients")

  ◦ Often supports handling numerator and denominator distinctly $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$ or $\frac{3}{4}x = 15$ interpreted as $\frac{3x}{4} = 15$

Society (Example from CBC)

○ Interestingly, A&W decided give the Quarter Pounder some competition in the 1980s. So it introduced the "Third-of-a-Pound Burger." It was priced the same as the Quarter Pounder but with a third of a pound of beef, instead of just a quarter pound. It even outperformed the Quarter Pounder in taste tests.

○ But nobody bought it.

○ When A&W did focus groups to try and figure out why, the reason was simple. And hilarious. It turns out, people aren't so savvy with fractions.

○ More than half the people in the focus groups questioned the price of the third-pounder. They wanted to know why they should have to pay the same price for a third of a pound as they did for a quarter pound at McDonald's. They said A&W was overcharging them. You're ripping them off.

CBC (Apr. 8, 2021). *How failing at fractions saved the Quarter Pounder*. Under the Influence.

Teacher Preparation

◦ One example of prospective EC-8 teachers (from Jones et al., 2019)

Item 27: $2x-3=7$	Item 28: $2x+\frac{1}{4}=10$			
	Correct	Incorrect	Blank	Total
Correct	136	202	60	398
Incorrect	5	15	11	31
Blank	0	1	39	40
Total	141	218	110	469

MPT

- In 2019 the Government put a math test in place as a requirement of becoming a certified teacher
- Applies to everyone who wishes to join the Ontario College of Teachers
  - Note that this applies if you get trained in Ontario but teach elsewhere.
- Fraction questions can randomly appear on the test, but they are limited in number and limited in scope
- Test is a proxy for some minimal capacity for teaching math
  - Has unfortunate consequences
    - high anxiety among all teacher candidates
    - Interferes with career start-up (particularly during pandemic)
    - We do not know who fraction issues correlate with

Research - Blue Sky

◦ Lenz et al. 2020 – procedural and conceptual details can be measured and separated

◦ "… strong evidence for the existence of a natural alignment between entity type and the format of rational numbers. This alignment, and other processing differences between fractions and decimals, cannot be attributed to the specifics of education, language, and measurement units, which differ greatly between the United States and South Korea." (Lee et al., 2016)

◦ Continued efforts to improve teaching –

  ◦ Trajectory research,
  ◦ Instructional methods (recognizing problematic approaches),
  ◦ Pre-service teachers – This is the largest area of research currently.

◦ Confidence based on familiarity rather than accuracy (Fitzsimmons, et al., 2020), Inhibition is regulated in response to counterintuitive fraction questions and cognitive load (Avgerinou & Tolmie, 2019).

High School – Some Performance Context

- 2001 – pharmacy entrance – 25% failed, further 25% under 70%

- 1998 NAEP Grade 4 (48% competence) and Grade 8 (64%)

- 1996 NAEP Grade 12 60% "mastery of simple fractions, decimals, percent and simple linear equations."

- 1995 NAEP $3\frac{1}{3} - 3\frac{1}{4}$ Grade 8 (35% a year later 36%)

- 1987 – Number between 6 and 6.1 – Grade 7-8:   38% with "can't be done" 49%

- 1976 NAEP
  - (1/2) + (1/3)   Grade 8 (42%) and Grade 12 (66%)
    - In 1999 was 33% in refresher course for nurses
 - (1/2)x(1/3)   Grade 8 (62%) and Grade 12 (74%)

High School – Ontario (Sibbald, 2005)

- Fraction and decimal performance do not change through grades 9, 10, and 11
- Performance on conceptual questions higher than procedural questions
- Overall fraction performance 38% sd=22% (approx.)
- Overall decimal performance 60% sd=27% (approx.)
- Overall mixed performance   39% sd=22% (approx.)
- Calculators improve, but not statistically significant

High School – Ontario (Sibbald, 2005) cont.

○ Questions connecting the two see some improvement

○ 2/5+0.3 39% compared to 1983 ¾+0.6 at 42%

○ Calculators improve, but not statistically significant

○ 22% acknowledged 3 or more meanings of a fraction

Opinions of utility (G 9, 10, 11)

Fractions 4.2%   Both 33.3%   Decimals 17.2%

Neither 9.0%

Don't know 35.8%
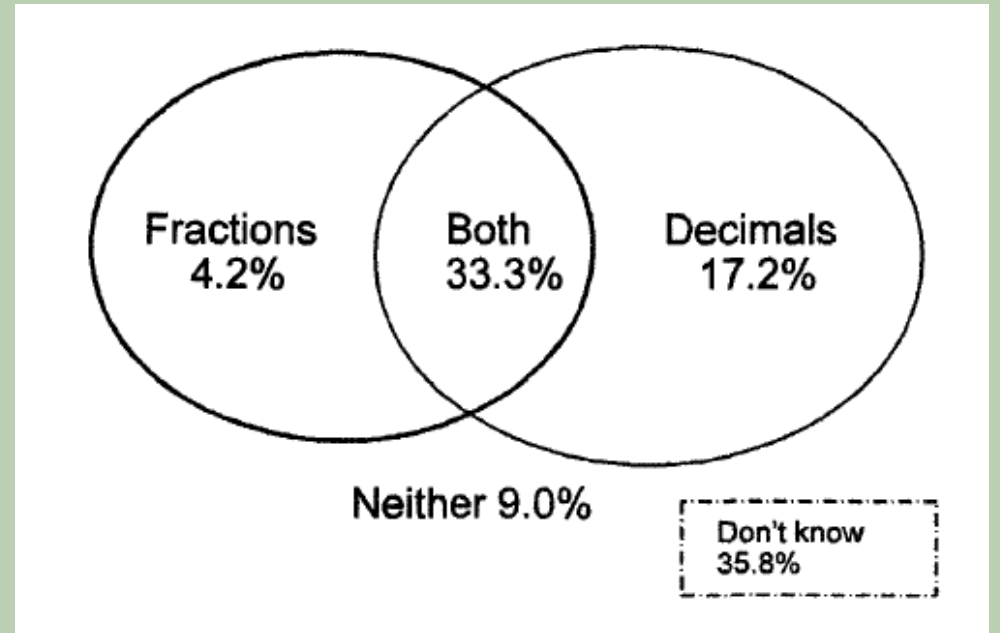
High School Realities

◦ High schools emphasize proper and improper fractions. Generally do not use mixed fractions.

◦ General migration to proportional reasoning as opposed to fraction skills

◦ Ratios get limited attention

◦ Remediation is usually in the form of compensation strategies

High School Strategies

◦ Compensation strategies?
  ◦ Emphasis on contextual sets of fractions
    ◦ Tape measure fractions – denominators as powers of two, using mixed fractions
    ◦ Baking fractions – denominator 6 or less.
      ◦ Sometimes developed to unit conversions for finer fractions 3tsp=tbsp., 2tbsp. = 1/8 cup
  ◦ Some use of conversion methods (Convert to decimals)
  ◦ Limited use of hands-on tools
  ◦ Limited additional instruction on basic skills

Colleges/Occupations - Research

◦ "college level students have the same misunderstandings as elementary school students regarding fraction operations" but they are more articulate when explaining their thinking. (Bentley & Bosse, 2018)

◦ College students emphasize procedural thinking over conceptual thinking. (Bentley & Bosse, 2018)

◦ "part-whole" difficulties estimated at 20% of college students (Steinke, 2015)

◦ The need to manage contextual details (Sredl, 2005; Rollings, 2019)

◦ Fractions used to filter students? – 25 000 FTE students: "… math proficiency, manifested in placement testing criteria, may unfairly and unnecessarily sort students to remedial coursework." / "of the students who missed the next level course by one point, 69% of those assigned to basic math instead of pre-algebra missed a fractions question, as did 45% of those assigned to pre-algebra instead of elementary algebra."

Nursing – A special case

◦ Nursing has invested a lot of time and effort with fractions – there is a long history of research, however, "There is currently no agreed benchmark setting out the vocational mathematics required for nursing education and professional practice" (Coben & Weeks, 2014)

◦ Calculators help but error rate still more than 1 in 10 with them. Programs to improve nursing students fractions typically struggle to achieve 85% accuracy

◦ Context and written are different – context with syringes, IV fluid bags, etc. (Coben & Weeks, 2014)

◦ "… that two skill are required. Firstly student nurses need to have basic mathematical skills in order to calculate mathematical problems and secondly student nurses need to be able to conceptualise clinical information presented and extract the relevant information in order to formulate a maths calculation to be solved" (Wright, 2007)

◦ "… on three major problem areas: multiplicity of mathematical formulas; students' inability to distinguish among and choose the right formula for a problem or situation; and verbalized feelings of professional ineptness." (Sredl, 2005)

Implications / Discussion

- Fraction performance was not "way better" years ago, and educators likely have a perception based on being a bias sample (academically success, old)

- If there was a performance decline it was 20 years ago, to make room for computers.

- Every aspect of fractions has longitudinal issues

- Evidence shows potential punitive consequences being loaded on fractions

- Fundamental systemic issues – but research is slow and not revealing solutions

- Occupational focus adds context and it can be a support or be a complicating factor – it is not a fix

- At what point do we change strategies?

References

Avgerinou, V. A., & Tolmie, A. (2019). Inhibition and cognitive load in fractions and decimals. *British Journal of Educational Psychology, 90*, 240–256.

Bentley, B., & Bosse, M. J. (2018). College students' understanding of fraction operations. *International Electronic Journal of Mathematics Education, 13*(3), 233–247. DOI 10.12973/iejme/3881

Coben, D., & Weeks, K. (2014). Meeting the mathematical demands of the safety-critical workplace: medication dosage calculation problems for nursing. *Educational Studies in Mathematics, 86*, 253–270. DOI 10.1007/s10649-014-9537-3

Fitzsimmons, C. J., Thompson, C. A., & Sydney, P. G. (2020). Confident or familiar? The role of familiarity rating in adults' confidence judgments when estimating fraction magnitudes. *Metacognition and Learning, 15*, 215–231. DOI: doi.org/10.1007/s11409-020-09225-9

Jones, D. L., Zientek, L. R., Sharon, V. V., & Swarthout, M. B. (2019). Solving equations with fractions: An analysis of prospective teachers' solution pathways and errors. *School Science and Mathematics, 120*, 232–243. DOI: 10.1111/ssm.12402

References – Cont.

Lee, H. S., DeWolf, M., Bassok, M., & Holyoak, K. J. (2016). Conceptual and procedural distinctions between fractions and decimals: A cross-national comparison. *Cognition, 147*, 57–69.

Lenz, K., Dreher, A., Holzapfel, L., & Wittmann, G. (2019). Are conceptual and procedural knowledge empirically separable? The case of fractions. *British Journal of Educational Psychology, 90*, 809–829.

Ngo, F. (2018). Fractions in college: How basic math remediation impacts community college students. *Research in Higher Education, 60*, 485–520.

Rollings, L. (2019). Drug calculations: A non-formulaic approach. *Mathematics Teaching, April*(266), 42–43.

Sredl, D. (2006). The triangle technique: A New Evidence-Based Educational Tool for Pediatric Medication Calculations. *Nursing Education Perspectives (National League for Nursing), 27*(2), 84–88.

Steinke, D. A. (2015). Evaluating number sense in workforce students. *Journal of Adult Education, 44*(1), 1–8.

Wright, K. (2007). Student nurses need more than maths to improve their drug calculating skills. *Nurse Education Today, 27*, 278–285.

Open Discussion

◦ I can be reached at timothys@nipissingu.ca

◦ And, yes, contrary to the research, I would welcome the opportunity to be involved in any research going on in this area.

◦ **Thank you, Derek Jennings, for managing the session**